

2. ALGEBRA

SYNOPSIS

- ★ Algebra is one of the most important branches of mathematics. In Algebra we start working with letters of alphabet as well as numbers and signs of operation. In Algebra we use lower case letters to represent numbers or quantities are called "Literals" or "Variables"
- ★ The process of computing various problems using variables is called "Algebra". The term Algebra is formed from the Arabic term "Al-Jabar"
- ★ Father of Algebra "Diophantus"
- ★ Constants (numbers) variables and operation symbols are combined to form mathematical expressions.
- ★ **Constant** : A number having a fixed numerical value is called a constant.
Ex. : 8, -9, 1, $\frac{-3}{4}$, 2.6 etc.
- ★ **Variable** : A number which can take various numerical values is known as variable.
Ex. a, b, c, x^2 , y, z, 4l etc.
- ★ **Term** : A term is a constant or variable and also it can be a product of constant and variables. A term can be either positive or negative.
Ex. : 4, -3xy, $\frac{7}{10}ab^2c$ etc.
- ★ **Expression** : An expression consists of one or more terms separated by '+' or '-'.
Ex. : $2x$, $3a-5$, $6x^2-7xy+1$ etc.
- ★ **Algebraic Expression** : The combination of constants and variables connected by '+' or '-' or '×' or '÷' signs is known as an Algebraic expressions.
Ex. : $2x+3y$, ax^2+bx+c etc.
- ★ **Coefficients** : Coefficients are two types.
 - a) Numerical coefficients
 - b) Literal (variable) coefficients
- a) **Numerical Coefficients** : In a term, the number factor of a product is the numerical coefficient of the remaining factors.
Ex. : In $64l^3$, the numerical coefficient of l^3 is 64.
(* When the numerical coefficient is not given, it is always understood as 1)

b) Literal Coefficients (Variable coefficient) :

The letter factor variable of a product is the "Literal Coefficient".

Ex. : In $9a^2b$, the literal coefficient is a^2b

In $2x$. The literal coefficient is x .

- ★ Like terms (Similar terms) : The terms having the same variable coefficients are called 'Like terms'.

Ex. : 1. $3x, \frac{-5}{2}x, 17x$ are like terms

2. $8a^2b, \frac{5}{7}a^2b, -11a^2b$ are like terms

- ★ Unlike terms (Disimilar terms) : The terms having different variable coefficients are called "Unlike terms"

Ex. : $x^2y, -6xy, 8xy^3$ are unlike terms.

★ Types of Algebraic Expressions:

a) Monomial : An expression containing only one term is called a "Monomial"

Ex. : $2x, 3a^2b, 3lmn$ etc.

b) Binomial : An expression containing two terms is called a "Binomial"

Ex. : $x - 2y, 2m^2n - 7, ax + b$

c) Trinomial : An expression containing three terms is called a "Trinomial"

Ex. : $x + y + z, ax^2 + bx + c, 3x^2 - 7x + 6$ etc.

- ★ Adding terms : Two or more Algebraic terms can be added if they are similar terms.

Ex. : 1. Add $2x$ and $4x$ ($\therefore 2x$ means $x + x$)

Sol. $2x + 4x = 6x$ $4x$ means $x + x + x + x$
total $6x$)

- ★ Subtracting terms : Two or more algebraic terms can be subtracted if they are similar terms.

Ex. : 1. Subtract $3x$ from $5x$

Sol. $5x - 3x = 2x$

- ★ Multiplying terms : When multiplying Algebraic terms it has to be performed as per the Index law of multiplication.

Ex. : 1. Multiply $2x$ and $3x$

Sol. $(2x) \times (3x) = 2 \times 3 \times x \times x = 6x^2$

2. Multiply $(x + 2)$ and $(2x + 3)$

Sol. $(x + 2)(2x + 3)$
 $= x(2x + 3) + 2(2x + 3)$
 $= 2x^2 + 3x + 4x + 6$
 $= 2x^2 + 7x + 6$

- ★ **Dividing Terms**: Two Algebraic terms can be divided by one with other and it necessary index law of division can be applicable.

Ex. : 1. $\frac{6x^2}{2x} = \frac{6.x.x}{2.x} = 3x$

2. $\frac{-8xy}{6x} = \frac{-4y}{3}$

Dividend = Divisor x Quotient + Remainder i.e. $p(x) = g(x).q(x) + r$
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- ★ **Polynomial** : An expression containing one or more monomials is called polynomial.

General form of polynomial is

$a + bx + cx^2 + dx^3 + \dots$, a, b, c, d are rational numbers and atleast one term is non zero.

- ★ **Degree of Polynomial**: The greatest of the degrees of the various terms is called the Degree of the Polynomial.

The general form of the 1st degree polynomial is

$$ax + b, (a \neq 0).$$

The general form of the 2nd degree polynomial is

$$ax^2 + bx + c (a \neq 0).$$

The general form of the nth degree polynomial is

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n (a \neq 0)$$

- ★ **Zero Polynomial** : If all the terms of the polynomial is zero then it is called "Zero Polynomial"

- ★ **Zero of the Polynomial** : The number for which the value of the polynomial is zero is called "Zero of the polynomial"

- ★ **Types of Polynomials** :

a) **Linear Polynomial** : A polynomial of degree 'one' is called a linear polynomial.

Ex. : $2x + 3$, $-6y + 4$ and $15a + 2$ are polynomials of degree one and hence they can be called as "Linear Polynomials"

b) **Quadratic Polynomial** : A polynomial of degree 'two' is called "Quadratic Polynomial"

Ex. : 1. $5x^2 + 7x + 1$ the degree is "2"

2. $ax^2 + bx + c$ the degree is "2"

c) **Cubic Polynomial** : A polynomial of degree 'three' is called "Cubic Polynomial"

Ex. : $2x^3 - 5x^2 + 6x + 2$ the degree is "3"
 d) Biquadratic Polynomial : A polynomial of degree 'four' is called "Biquadratic Polynomial"

Ex. : $8x^4 - 7x^3 + 5x + 3$ the degree is "4"
 e) Constant Polynomial : A polynomial having no term which is a constant is called "Constant Polynomial"
 The degree of Constant Polynomial is "0"

Ex. : $2, 7, \frac{-4}{3}$ degree is '0'

★ Numerical sentences : A numerical sentence is formed by two numerical expressions combined with $=, >, <, \neq, \nless, \ngtr$ etc.

Ex. : $5 + 4 = 9, 3 - 1 < 4$

★ Numerical statement : A numerical sentence is said to be a statement, if the sentence is either true or false.

Ex. : $4 + 3 = 7 \Rightarrow \text{True}$

$8 + 2 = 11 \Rightarrow \text{False}$

★ Mathematical sentence : A mathematical sentence is formed by an algebraic expression and a numerical expression (or) two algebraic expressions combined with one of the following relations $=, \neq, >, <, \nless, \ngtr$

Ex. : $2x + 5 = 17, 2x^2 - 3x > 10$

★ Open sentence : The sentence whose truth value cannot be determined is called an open sentence.

Ex. : $x^2 \geq 0$ is always true.

★ Equation : An open sentence with equality sign is called an Equation.
 Ex. : $2x + 4 = 12$ here $2x + 4$ is called L.H.S. 10 is called R.H.S.

★ Identity : An Identity is an equation which is true for all the values of variables.

Ex. : 1. $2(a + b) = 2a + 2b$

2. $(a + b)^2 = a^2 + 2ab + b^2$

★ Identities also called special products :

1. $(a + b)^2 = a^2 + 2ab + b^2$

2. $(a - b)^2 = a^2 - 2ab + b^2$

3. $a^2 - b^2 = (a + b)(a - b)$

4. $a^2 + b^2 = (a + b)^2 - 2ab$ or $(a - b)^2 + 2ab$
8. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
13. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ or $a^3 + 3ab(a + b) + b^3$
14. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ or $(a + b)^3 - 3a^2b - 3ab^2$
15. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ or $a^3 - 3ab(a - b) - b^3$
16. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$ or $(a - b)^3 + 3a^2b - 3ab^2$
17. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
20. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
22. $\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$
- * If $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$
- * If $\left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$
- * If $x + \frac{1}{x} = a$ then $x^2 + \frac{1}{x^2} = a^2 - 2$
- * If $x - \frac{1}{x} = a$ then $x^2 + \frac{1}{x^2} = a^2 + 2$
- * If $x^{a+1} + \frac{1}{x^{a+1}} = \left(x^a \frac{1}{x^a}\right)\left(x + \frac{1}{x}\right) - \left(x^{a-1} + \frac{1}{x^{a-1}}\right)$
- * If $xy + yz + zx = xyz$ then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$
- * If $a + b + c = 0$ or $a = b = c$ then $a^3 + b^3 + c^3 = 3abc$
- * $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$
- * If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$
- * $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$
- * $(x + a)(x + b) = x^2 + (a + b)x + ab$
- * $(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$

- * $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$
- * **Equality** : A true mathematical statement containing the sign is “is equal to” is called an “Equality”
Ex. : $9+4=13$, $10+15=25$
- * **Roots of the equation** : A number which satisfies an equation is called “Solution” or “roots of the equation”
Ex. : $x+4=9 \Rightarrow x=9-4=5$
If $x=5$, $x+4=9$ is true
 $\therefore$ ‘5’ is the root of $x+4=9$
- * **Linear Equation** : An equation in which the highest index of the variable present is one is called a “Linear Equation”
Ex. : $x+y=4$, $a+4=23$
- * **Simple Equation** : A linear equation in one variable is called “Simple Equation”
Ex. : $x+1=4$, $a+2=7$
- * **Transposition** : Changing a term from one side of an equation to the other side is called Transposition.
Ex. : $+\rightarrow-$, $-\rightarrow+$, $\times\rightarrow\div$, $\div\rightarrow\times$
- * **Inequation** : An open sentence which consists of one of the symbols $>$, $<$, $\geq$, $\leq$, is called inequations.
Ex. : i) $5x-4>12$ ii) $3p+6<32$
- * **Continued Inequation**: Two inequations of the same type (both consisting of “ $>$ or $\geq$ ” or both consisting of “ $<$ or $\leq$ ”) can be combined into a continued inequation.
Ex. : i) If $a<b$ and $b<c$, we can write $a<b<c$
ii) If $a\geq b$ and $b>c$, we can write $a\geq b>c$
- * **Linear Equations** : An inequation in which the highest degree of the variable present is one is called a “Linear inequation”
Ex. i) $5x-3\leq 10-3x$
ii) $2x+10\geq x+5$
- * Equations which are used frequently to solve problems are called “Formulae”
Ex. : $A = \frac{1}{2}d_1d_2$ is used to find Area of Rhombus.
- * ‘The area of a rectangle is the product of its length and breadth’ statement symbolic form is $A = 'lb'$ where A is Area, l is length, b is breadth.
- * In a $m \times n$ network, we find “mn” squares and $(m+1)(n+1)$ points.
- * If a polygon consists ‘n’ sides then the number of diagonals are : $\frac{n(n-1)}{2} - n$

- ★ The number of squares contained in a $K \times K$ network are $1^2 + 2^2 + 3^2 + \dots + K^2$
- ★ Triangular numbers : 1,3,6,10,15..... $\frac{n(n+1)}{2}$
- ★ $1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2} = \sum n = \frac{n^2}{2} + \frac{n}{2} \equiv$
- ★ $1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} = \sum n^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$
- ★ $1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2 = \frac{n^2(n+1)^2}{4} = (\sum n)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- ★ In a^n 'a' is called 'base' and 'n' is called an 'exponent'
Ex. : $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$ (2 is base ; 6 is Index)
- ★ The first power of a number is the number itself i.e. $x^1 = x$
- ★ The second power is called "Square" and the third power is called "Cube"
Ex. : Square of 5 is 5^2 Cube of 5 is 5^3
- ★ '1' raised to any integral power gives '1'
Ex. : $1^{7102011} = 1$
- ★ i) When '-1' is raised to an odd positive integral power, it gives '-1'.
Ex. : $(-1)^{20100711} = -1$
- ii) When '-1' is raised to an even positive integral power, it gives '1'
Ex. : $(-1)^{980} = 1$

Laws of Exponents :

- i) $a^m \times a^n = a^{m+n}$
- ii) $(a^m)^n = a^{mn}$
- iii) $(ab)^m = a^m b^m$
- iv) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- v) $\frac{a^m}{a^n} = a^{m-n}$, (if $m > n$)
 $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, (if $m < n$)
 $\frac{a^m}{a^n} = 1$, (if $m = n$)
- vi) $a^n = \frac{1}{a^{-n}}$, $a^{-n} = \frac{1}{a^n}$

vii) $a^0 = 1$, where 'a' is a non zero real number (Note : We will not let $0^0 = 1$, because 0^0 can lead to division by 0. i.e. $0^0 = 0^{n-n} = \frac{0^n}{0^n}$ thus 0^0 is not defined.)

$$\star \sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

$$\star \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\star (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$$

$$\star (\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$$

$$\star (\sqrt{a} - \sqrt{b})^2 = a - 2\sqrt{ab} + b$$

$$\star (a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$$

$\star$ If the bases are equal then powers must be equal.

Ex. : If $2^n = 2^6$ then $n = 6$

$\star$ If the powers are equal then the bases must be equal.

Ex. : If $7^{10} = m^{10}$ then $m = 7$

$\star a^{\frac{1}{n}} = \sqrt[n]{a}$ called the n^{th} root of a.

$$\star \sqrt[n]{a^n} = a^{\frac{n}{n}} = a^1 = a$$

$\star \sqrt[n]{a^m} = a^{\frac{m}{n}}$ called n^{th} root of a^m .

$$\star \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$$\star \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\star \sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$$

$$\star \sqrt[m]{a^n} = (\sqrt[m]{a})^n$$

$\star$ **Scientific Notation** : In a Scientific Notation as a product of two factors one factor is a number between 1 and 10. The second factor is a power of 10.

a) For the first factor move the decimal point to the left as many places as it takes to get a number between 1 and 10.

b) Next, use the number of places moved as the power of 10 for the second factor.

Ex. : The distance between the earth and the sun is 149,600,000,000 mts. write in scientific notation.

Sol. : The given number can be defined by scientific

notation as 1.496×10^{11} mts.

- ★ The standard form is to think of the decimal point moving to the right. The power of 10 tells you how many places to move it.

Ex. : $1.496 \times 10^{11} = 149,600,000,000$ (11 places)

- ★ The product of monomial and multinomial, multiply each term of the multinomial by the monomial.

Ex. : $(x^2 + 6x + 5) \times (-7x)$
 $= x^2(-7x) + 6x(-7x) + 5(-7x)$
 $= -7x^3 - 42x^2 - 35x$

- ★ Every monomial which is a perfect square, has two square roots of these the positive square root is called the "Principal Square root".
- ★ Method of Factorization of quadratic expressions :
 - a) Multiply the coefficient of x^2 by the constant term.
 - b) Resolve this product into two factors such that their sum is the coefficient of x
 - c) Rewrite the x term as the sum of two terms with these coefficients.
 - d) Group them into two parts each containing two terms and factorize.

Ex. : Factorize $x^2 - 5x + 6$

Sol. : $x^2 - 5x + 6$ Here coefficient of x^2 is 1 and constant term 6

so $1 \times 6 = 6$

The factors of 6 = 1×6 or -1×-6

= 2×3 or -2×-3 it can be chosen

$$\begin{aligned} &= x^2 - 5x + 6 \\ &= x^2 - 2x - 3x + 6 \\ &= x(x - 2) - 3(x - 2) \\ &= (x - 2)(x - 3) \end{aligned}$$

PROBLEMS

1. Solve $5x - 8 = 3x + 22$

Sol. $5x - 8 = 3x + 22$
 $\Rightarrow 5x - 3x = 22 + 8$
 $\Rightarrow 2x = 30$

$$x = \frac{30}{2}$$

$$x = 15$$

2. Simplify $-2x^2 - (-x^2 + 4x)$

$$\begin{aligned}
 \text{Sol. } & -2x^2 - (-x^2 + 4x) \\
 & \Rightarrow -2x^2 + x^2 - 4x \\
 & \Rightarrow -x^2 - 4x \\
 & \Rightarrow -(x^2 + 4x)
 \end{aligned}$$

3. Find the value of $(1+x^{92})(1-x^{141})$ when $x = -1$

$$\begin{aligned}
 \text{Sol. } & \text{If } x = -1 \text{ then } (1+x^{92})(1-x^{141}) \\
 & = (1+(-1)^{92})(1-(-1)^{141}) \\
 & = (1+1)(1-(-1)) \\
 & = (1+1)(1+1) \\
 & = 2 \times 2 \\
 & = 4
 \end{aligned}$$

4. If a,b,c are three real numbers, $a - 7b + 8c = 4$ and $8a + 4b - c = 7$ then the value of $a^2 - b^2 + c^2 = \dots\dots\dots$

$$\begin{aligned}
 \text{Sol. } & \text{i) } a - 7b + 8c = 4 \\
 & a + 8c = 4 + 7b \text{ (sq.on bothsides)}
 \end{aligned}$$

$$\Rightarrow a^2 + 16ac + 64c^2 = 16 + 56b + 49b^2 \rightarrow (1)$$

$$\begin{aligned}
 & \text{ii) } 8a + 4b - c = 7 \\
 & (8a - c) = (7 - 4b) \text{ (sq.on bothsides)}
 \end{aligned}$$

$$\begin{aligned}
 64a^2 + c^2 - 16ac &= 49 + 16b^2 - 56b \rightarrow (2) \\
 & \text{adding (1) and (2)}
 \end{aligned}$$

$$a^2 + 16ac + 64c^2 = 16 + 56b + 49b^2$$

$$64a^2 - 16ac + c^2 = 49 - 56b + 16b^2$$

$$\Rightarrow 65a^2 + 65c^2 = 65 + 65b^2$$

$$\Rightarrow 65(a^2 + c^2) = 65(1 + b^2)$$

$$\Rightarrow a^2 + c^2 = 1 + b^2$$

$$\therefore a^2 - b^2 + c^2 = 1$$

5. If $P = 3^{2018} + 3^{-2018}$ and $Q = 3^{2018} - 3^{-2018}$ then $P^2 - Q^2 = \dots\dots\dots$

Sol. $P = 3^{2018} + 3^{-2018}$ is the form of

$$P = 3^{2018} + \frac{1}{3^{2018}}$$

If $3^{2018} = a$ let

$$P = a + \frac{1}{a} \Rightarrow P^2 = a^2 + 2 + \frac{1}{a^2}$$

$$\text{Similarly } Q = a - \frac{1}{a} \Rightarrow Q^2 = a^2 - 2 + \frac{1}{a^2}$$

$$\therefore P^2 - Q^2 = 4$$

6. If $8^{x+2} = 2^{4x-3}$ then $x = \dots\dots\dots$

Sol. $8^{x+2} = 2^{4x-3}$

$$\Rightarrow (2^3)^{x+2} = 2^{4x-3}$$

$$\Rightarrow 2^{3x+6} = 2^{4x-3}$$

Base are equal so exponents are equal

$$3x + 6 = 4x - 3$$

$$4x - 3x = 6 + 3$$

$$\therefore x = 9$$

7. The sum of the digits of a two digit number is 9. If 27 is subtracted from the number, the digits interchange their places. Find the numbers.

Sol. Let the digit in the unit place be x

the digit in the tens place is $9 - x$ ($\because$ sum is 9)

$$\text{the number} = 10(9 - x) + x = 90 - 10x + x = 90 - 9x$$

The number formed by interchanging the digits = $10x + 9 - x = 9x + 9$

According to data here $9x + 9 = 90 - 9x - 27$

$$\Rightarrow 9x + 9x = 90 - 9 - 27$$

$$18x = 54$$

$$x = \frac{54}{18}$$

$$x = 3$$

$\therefore$ Unit digit = 3

Tens digit = $9 - 3 = 6$

$\therefore$ The number = 63

8. Sixteen years hence a man's age will be 9 times his age 16 years ago. Find his age 5 years hence.

Sol. Let present age = x years

According to data

$$\Rightarrow x + 16 = 9(x - 16)$$

$$\Rightarrow x + 16 = 9x - 144$$

$$\Rightarrow 9x - x = 16 + 144$$

$$\Rightarrow 8x = 160$$

$$\Rightarrow x = \frac{160}{8}$$

$$x = 20$$

Age 5 years hence will be = $x + 5$

$$= 20 + 5$$

$$= 25 \text{ years}$$

9. The value of $\frac{3^8(11111)^4}{(33333)^4}$ is

Sol. $\frac{3^8(11111)^4}{(33333)^4} = \frac{3^4 \cdot 3^4 (11111)^4}{(33333)^4} = \left(\frac{3 \cdot 3 \cdot 11111}{33333} \right)^4 = 3^4 = 81$

10. If $3^n = 27$ then 3^{n-2} is

Sol. $3^n = 27$

$$3^{n-2} = \frac{3^n}{3^2} = \frac{27}{9} = 3 \quad \left(\because a^{m-n} = \frac{a^m}{a^n} \right)$$

11. Solve $\frac{3x}{2} + \frac{x}{3} = x + \frac{x}{6} + 6$

Sol. $\frac{3x}{2} + \frac{x}{3} = x + \frac{x}{6} + 6$

$$\Rightarrow \frac{3x}{2} + \frac{x}{3} - x - \frac{x}{6} = 6$$

$$= \frac{9x + 2x - 6x - x}{6} = 6$$

$$\Rightarrow 11x - 7x = 36$$

$$\Rightarrow 4x = 36$$

$$x = \frac{36}{4}$$

$$x = 9$$

12. The sum of two consecutive odd numbers is 164. Find the numbers.

Sol. Let the two consecutive odd numbers be x and $x + 2$

Their sum is $x + (x + 2) = 164$

$$\Rightarrow 2x + 2 = 164$$

$$\Rightarrow 2x = 164 - 2$$

$$\Rightarrow 2x = 162$$

$$\Rightarrow x = \frac{162}{2}$$

$$x = 81$$

The numbers are $x = 81$

$$x + 2 = 81 + 2 = 83$$

The numbers 81 and 83

13. If $x^2 + \frac{1}{x^2} = 2$ then $x^2 - \frac{1}{x^2} = \dots\dots\dots$

Sol. $x^2 + \frac{1}{x^2} = 2 \Rightarrow x^4 + 1 = 2x^2$

$$\Rightarrow x^4 - 2x^2 + 1 = 0$$

$$\Rightarrow (x^2)^2 - 2 \cdot x^2 - 1 + 1^2 = 0$$

$$\Rightarrow (x^2 - 1)^2 = 0$$

$$\Rightarrow x^2 - 1 = 0$$

$$\Rightarrow x^2 - 1$$

$$x = \pm 1$$

$$\text{If } x = 1 \text{ then } x^2 - \frac{1}{x^2} = (1)^2 - \frac{1}{(1)^2} = 1 - \frac{1}{1} = 1 - 1 = 0$$

$$\text{If } x = -1 \text{ then } x^2 - \frac{1}{x^2} = (-1)^2 - \frac{1}{(-1)^2} = 1 - \frac{1}{1} = 1 - 1 = 0$$

$$\therefore x^2 + \frac{1}{x^2} = 2 \text{ then } x^2 - \frac{1}{x^2} = 0$$

14. The number of diagonal of a regular polygon having 20 sides

Sol. The number of diagonal of a regular polygon having

$$\text{'n' sides} = \left[\frac{n(n-1)}{2} - n \right]$$

$$\begin{aligned} 20 \text{ sides} &= \left[\frac{20(20-1)}{2} - 20 \right] \\ &= 10 \times 19 - 20 \\ &= 190 - 20 \\ &= 170 \end{aligned}$$

15. If a and b are two positive integers such that $a^b = 125$ then $(a-b)^{a+b-4} = \dots\dots$

$$\text{Sol. } a^b = 125 = 5^3 \Rightarrow a = 5$$

$$b = 3$$

$$(a-b)^{a+b-4} = (5-3)^{5+3-4} = 2^{8-4} = 2^4 = 16$$

16. $5\sqrt{5} \times 5^3 \div 5^{\frac{-3}{2}} = 5^{a+2}$ then the value of 'a'

$$\text{Sol. } 5\sqrt{5} \times 5^3 \div 5^{\frac{-3}{2}} = 5^{a+2}$$

$$\Rightarrow 5^1 \cdot 5^{\frac{1}{2}} \times \frac{5^3}{5^{\frac{-3}{2}}} = 5^{a+2} \quad \left(\because \frac{1}{a^{-n}} = a^n \right)$$

$$\Rightarrow 5^{\frac{3}{2}} \times 5^3 \times 5^{\frac{3}{2}} = 5^{a+2}$$

$$\Rightarrow 5^{\frac{3}{2}+3+\frac{3}{2}} = 5^{a+2}$$

$$\Rightarrow \frac{3}{2} + 3 + \frac{3}{2} = a + 2$$

$$\Rightarrow \frac{3+6+3}{2} = a + 2$$

$$\Rightarrow \frac{12}{2} = a + 2$$

$$\therefore a + 2 = 6$$

$$a = 6 - 2$$

$$a = 4$$

17. The value of $\frac{(2018^2 - 2024)(2018^2 + 4036 - 3)2019}{(2015)(2017)(2020)(2021)} = \dots\dots$

Sol.
$$\frac{((2018^2 - 9) - 2015)(2018^2 + 2 \cdot 2018 + 1^2 - 4)2019}{2015 \cdot 2017 \cdot 2020 \cdot 2021}$$

$$= \frac{[(2018 + 3)(2018 - 3) - 2015][(2018 + 1)^2 - 2^2]2019}{2015 \cdot 2017 \cdot 2020 \cdot 2021}$$

$$= \frac{(2021 \cdot 2015 - 2015)(2019 + 2)(2019 - 2) \cdot 2019}{2015 \cdot 2017 \cdot 2020 \cdot 2021}$$

$$= \frac{2015(2021 - 1)2021 \cdot 2017 \cdot 2019}{2015 \cdot 2017 \cdot 2020 \cdot 2021}$$

$$= \frac{2020 \cdot 2019}{2020} = 2019$$

18. If the product of two numbers is 21 and their difference is 4 then the ratio of the sum of their cubes of the difference of their cubes is

Sol. Let the two numbers be x and y

The product of x and $y = xy = 21$

Their difference $= x - y = 4 \rightarrow$ (1)

$$= (x + y)^2 = (x - y)^2 + 4xy$$

$$= (4)^2 + 4(21)$$

$$= 16 + 84$$

$$= 100$$

$$\therefore x + y = \sqrt{100}$$

$$x + y = 10 \rightarrow$$
 (2)

From (1) and (2)

$$x + y = 10$$

$$x - y = 4$$

$$2x = 14$$

$$x = \frac{14}{2}$$

$$x = 7$$

$\therefore$ The numbers are 7 and 3

the sum of the cubes $= 7^3 + 3^3 = 343 + 27 = 370$

The difference of the cubes $= 7^3 - 3^3 = 343 - 27 = 316$

Their ratio $= 370 : 316$

$= 185 : 158$

19. If $a^x = \sqrt{b}$, $b^y = \sqrt[3]{c}$ and $c^z = \sqrt{a}$ then find xyz

Sol: $a^x = \sqrt{b}$ $b^y = \sqrt[3]{c}$

$$a^x = (b)^{1/2} \quad b^y = (c)^{1/3}$$

$$a = (b)^{\frac{1}{2x}} \quad b = c^{\frac{1}{3y}}$$

Given $c^z = \sqrt{a}$

$$c^z = (a)^{\frac{1}{2}}$$

$$c^z = \left(b^{\frac{1}{2x}} \right)^{\frac{1}{2}} \left(\left(c^{\frac{1}{3y}} \right)^{\frac{1}{2x}} \right)^{\frac{1}{2}} = c^{\frac{1}{12xy}}$$

$$z = \frac{1}{12xy} \Rightarrow 12xyz = 1$$

$$xyz = \frac{1}{12}$$

20. If $x = 5 + 2\sqrt{6}$ then find $x + \frac{1}{x}$

Sol: $x = 5 + 2\sqrt{6}$

$$\frac{1}{x} = \frac{1}{5 + 2\sqrt{6}} \times \frac{5 - 2\sqrt{6}}{5 - 2\sqrt{6}} = \frac{5 - 2\sqrt{6}}{(5)^2 - (2\sqrt{6})^2} = \frac{5 - 2\sqrt{6}}{25 - 24} = 5 - 2\sqrt{6}$$

$$x + \frac{1}{x} = 5 + 2\sqrt{6} + 5 - 2\sqrt{6} = 10$$

21. If $a^3 + 3a^2 + 3a + 2 = 0$ then find $(a+1)^{2018} + (a+1)^{2019} + (a+1)^{2020}$

Sol: $a^3 + 3a^2 + 3a + 2 = 0$

$$a^3 + 3a^2 + 3a + 1 + 1 = 0$$

$$(a+1)^3 + 1 = 0$$

$$(a+1)^3 = -1 \Rightarrow a+1 = -1$$

$$(a+1)^{2018} + (a+1)^{2019} + (a+1)^{2020} = (-1)^{2018} + (-1)^{2019} + (-1)^{2020}$$

$$= 1 + (-1) + 1$$

$$= 2 - 1 = 1$$

22. If $x = a(b - c)$, $y = b(c - a)$ and $z = c(a - b)$ then find $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$

Sol: $\frac{x}{a} = b - c$ $\frac{y}{b} = c - a$ $\frac{z}{c} = a - b$

$$\begin{aligned} \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 &= (b - c)^3 + (c - a)^3 + (a - b)^3 & (a + b + c = 0 \text{ then} \\ &= 3(a - b)(b - c)(c - a) & a^3 + b^3 + c^3 = 3abc) \\ &= 3 \cdot \frac{z}{c} \cdot \frac{x}{a} \cdot \frac{y}{b} = \frac{3xyz}{abc} \end{aligned}$$

23. If $x = \frac{154}{69}$ then find $(c + d) - (a + b) = \dots\dots\dots$

Sol: $\frac{154}{69} = 2 + \frac{16}{69} = 2 + \frac{1}{\frac{69}{16}} = 2 + \frac{1}{4 + \frac{5}{16}} = 2 + \frac{1}{4 + \frac{1}{\frac{16}{5}}}$

$$= 2 + \frac{1}{4 + \frac{1}{3 + \frac{1}{5}}}$$

$a = 2, b = 4, c = 3, d = 5$

$(c + d) - (a + b) = (3 + 5) - (2 + 4) = 8 - 6 = 2$

24. If $x^2 + xy = 20$ and $y^2 + xy = 30$ then find xy

Sol: $x^2 + xy + y^2 + xy = 20 + 30$

$x^2 + 2xy + y^2 = 50$

$(x + y)^2 = 50$

No $(x^2 + xy)(y^2 + xy) = 20 \times 30$

$x(x + y)y(x + y) = 600$

$xy(x + y)^2 = 600$

$xy(50) = 600$

$xy = \frac{600}{50} \quad xy = 12$

PROBLEMS

LEVEL - 1

1. The degree of the polynomial $3x^7 + 5x^6 - 2x^5 + 10x^2$ is []
 A) 8 B) 7 C) 6 D) 3
2. $2x - 2\left(x - \frac{1}{2}\right) = \dots\dots\dots$ []
 A) x B) $2x$ C) 2 D) 1
3. If the sum of predecessor and the successor of a number is 116 then the number is []
 A) 58 B) 59 C) 57 D) 60
4. If $A = S^2$ and $A = 324 \text{ cm}^2$ then the value of $S = \dots\dots\dots$ []
 A) 162 B) $(324)^2$ C) 18 D) 16
5. Which of the following is a solution of $2x - 5 > 4x - 3$? []
 A) 0 B) 1 C) -1 D) -2
6. If $3(4m + 5) - 4(3 - 2m) = 13$ then $m = \dots\dots\dots$ []
 A) $\frac{1}{2}$ B) 2 C) $\frac{1}{4}$ D) 1
7. If $x = 2$ and $y = 4$ then $x^{\frac{x}{y}} \times y^{\frac{x}{y}} = \dots\dots\dots$ []
 A) 2 B) 4 C) $2^{\frac{2}{3}}$ D) $2^{\frac{3}{2}}$
8. The value of $(-1)^0 + (-1)^1 + (-1)^2 + \dots\dots + (-1)^{100} = \dots\dots\dots$ []
 A) 0 B) 1 C) 2 D) 4
9. $(2^3)^4 = \dots\dots\dots$ []
 A) 2^{4^3} B) (2^{3^4}) C) $(2^4)^3$ D) 2^7
10. If $a + b + c = 0$ then the value of $\sqrt{x^a \cdot x^b \cdot x^c} = \dots\dots\dots$ []
 A) 0 B) 1 C) -1 D) x^3
11. If $3^x = 6561$ then $3^{x-3} = \dots\dots\dots$ []
 A) 81 B) 243 C) 729 D) 27
12. If $(5^0 + 6^0) \times 2^0 - 7^0 = \dots\dots\dots$ []
 A) 0 B) 11 C) 15 D) 1
13. $(a-1)(a^2 - 2a + 1) = \dots\dots\dots$ []
 A) $(a-1)^2$ B) $(a-1)^3$ C) $a^2 - 1$ D) 1
14. If $y + \frac{1}{2y} = 4$ then $y^2 + \frac{1}{4y^2} = \dots\dots\dots$ []
 A) 15 B) 16 C) 18 D) 10
15. The zero of the $2x + 3$ is []
 A) -2 B) -3 C) $-\frac{3}{2}$ D) $-\frac{2}{3}$

16. If $496 \times 492 = x^2 - 4$ ($x > 0$) then $x = \dots\dots$ []
 A) 495 B) 494 C) 493 D) 496
17. $46 \times 46 + 54 \times 54 + 2 \times 46 \times 54 = \dots\dots$ []
 A) 9996 B) 10004 C) 9800 D) 10000
18. The perimeter of a rectangle whose length is $8a - 3b$ and breadth is $5a - 6b$ []
 A) $26a - 18b$ B) $24a + 18b$ C) $26a + 18b$ D) $22a - 16b$
19. If $x = 6, y = 12, z = -3$ then $\frac{x}{2} + \frac{y}{4} - \frac{z}{3} = \dots\dots$ []
 A) 2 B) 3 C) 6 D) 7
20. If $a^{y-x} \cdot b^{z-y} \cdot c^{x-z} \cdot d^{x+y+z}$, if a, b, c, d are the first four powers of 3 and x, y, z are the first 3 natural numbers, then the monomial = []
 A) 3^{30} B) 3^{33} C) 3^{32} D) 3^{34}
21. If $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $a = 85, b = 40, c = 75$ and $s = \frac{1}{2}(a+b+c)$ then $A = \dots\dots$ []
 A) 1475 B) 1516 C) 1500 D) 1600
22. If $x = 2, y = 1.15, z = 0.01$ then the value of $xy(x^2 + y^2)$ is []
 A) 0.10276 B) 0.1058 C) 12.24175 D) 0.12572
23. The degree of $3x^2y^5z^4$ is []
 A) 2 B) 11 C) 4 D) 5
24. $(256)^{0.16} \times (256)^{0.09} = \dots\dots$ []
 A) 256 B) 64 C) 16 D) 4
25. $[1 + 2^2 + 3^3 + 4^4 + \dots + 100^{100}]^p = \dots\dots$ []
 A) 0 B) 1 C) 100 D) 100^{100}
26. The value of $\left\{\left(\frac{-4}{3}\right)^{-3}\right\}^2$ is []
 A) $\frac{3}{4}$ B) $-\frac{3}{4}$ C) $\frac{1024}{243}$ D) $\frac{729}{4096}$
27. $(1 + x)^{1+x^2+2x} = 4^{16}$ then value of 'x' []
 A) 1 B) 2 C) 3 D) 25
28. $\frac{8^8(11111)^4}{(88888)^4} = \dots\dots$ []
 A) 2796 B) 4396 C) 4096 D) 1824
29. $a^{x-y+1} \cdot a^{y-z+1} \cdot a^{z-x+1} = \dots\dots$ []
 A) a^3 B) a^4 C) a D) a^2
30. The value of $[1^3 + 2^3 + 3^3 + 4^3]^{\frac{3}{2}} = \dots\dots$ []

- A) 4000 B) $\frac{1}{4000}$ C) 100 D) $\frac{1}{1000}$
31. If $8^{x+2} = 2^{4x-3}$ then $x = \dots\dots$ []
 A) 5 B) 9 C) 7 D) 6
32. If $m=3$ and $n=2$ then $2m^3 + 3n^2 - 5m^2n = \dots\dots$ []
 A) -24 B) 26 C) 24 D) -26
33. If $x^y = y^x$ then $\left(\frac{x}{y}\right) = \dots\dots$ []
 A) $\frac{x}{y}$ B) $\frac{x-y}{y}$ C) $\frac{y}{x^x}$ D) $\frac{y-x}{x}$
34. Express 0.0084 in scientific notation []
 A) 84×10^{-4} B) 8.4×10^{-3} C) 0.84×10^{-2} D) 840×10^{-1}
35. The value of 'a' of expression $3x^2 + 4x + a = 18$, when $x = -1 = \dots\dots$ []
 A) 17 B) 18 C) 19 D) 20
36. $\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b = \dots\dots$ []
 A) 0 B) 1 C) 2 D) 3
37. If $x=6$, $y=12$, $z=-3$ then the value of $\frac{3x}{z} + \frac{y}{4} - \frac{z^2}{3} = \dots\dots$ []
 A) 2 B) 3 C) 6 D) 9
38. If $p+q+r=124$, $p-q=26$ and $q-r=7$ then one of the following order of p,q,r is []
 A) 35,61,28 B) 61,28,35 C) 28,35,61 D) 61,35,28
39. If $7 - \{3x - (4x - 5) - 3\} = 0$ then the value of $x = \dots\dots$ []
 A) -3 B) -4 C) -6 D) -5
40. Divide 'x' by '4' and then add 13, if the result is 33. then $x = \dots\dots$ []
 A) 20 B) 46 C) 80 D) 113
41. Equation form of the product of the square of p and 3 more than 4 times p is []
 A) $p^2 \times (4p+3)$ B) $p^2 \times (3p+4)$ C) $p^2 \times (4p-3)$ D) $p^2 \times (3p-4)$
42. If $\frac{m}{5} = 6$ and $p-4=8$ then the value of $m+p = \dots\dots$ []
 A) 43 B) 41 C) 42 D) 44
43. If $c = \frac{5}{9}(F-32)$ then making F as subject we get []
 A) $F = \frac{5}{9}c + 32$ B) $F = \frac{9}{5}c - 32$ C) $F = \frac{9}{5}c \pm 32$ D) $F = \frac{9}{5}c + 32$
44. The sum of 4 consecutive integers is 70. Then greatest among them is .. []
 A) 19 B) 23 C) 17 D) 16

45. How many pieces $3\frac{4}{7}cm$ long can be cut from a long type. Which is 1m 75 cm
 A) 49 B) 50 C) 51 D) 52
46. If $a - b = 2$ then $a^2 + 2b - 4$ in terms of $b = \dots\dots$ []
 A) $b^2 - 6b + 4$ B) $b^2 + 4b$ C) $b^2 + 6b$ D) $b^2 + 6b - 4$
47. If $A = \frac{1}{4} + \left(\frac{1}{5}\right)^{-1}$ and $B = \frac{1}{5} + \left(\frac{1}{4}\right)^{-1}$ then $A + B = \dots\dots$ []
 A) $\frac{197}{20}$ B) $\frac{189}{20}$ C) $\frac{191}{20}$ D) $\frac{193}{20}$
48. How many zeroes are present in the cube of the number 25000 []
 A) 27 B) 9 C) 6 D) 25
49. The equation $2x + 1 = 5$ is identical to []
 A) $3x + 1 = 5$ B) $x = 4$ C) $6x + 3 = 15$ D) $4x + 2 = 16$
50. If $x + \frac{1}{x} = 6$ then $x^2 + \frac{1}{x^2} = \dots\dots$ []
 A) 34 B) 36 C) 32 D) 38
51. Degree of constant term = []
 A) 1 B) 2 C) 0 D) Not defined
52. $A = x^2 + x + 1$, $B = x^2 - x + 1$ then $A + B = \dots\dots$ []
 A) $2x^2 + 1$ B) $2x^2 + 2$ C) $2x^2 - 2x + 2$ D) $2x^2 + 2x + 1$
53. Additive inverse of $x + 2$ is []
 A) $2x$ B) $-x + 2$ C) $x - 2$ D) $-x - 2$
54. $P = 3x - 4y - 8z$, $Q = -10y + 7x + 11z$ and $R = 19z - 6y + 4x$ then $P - Q + R =$ []
 A) $13x - 20y + 16z$ B) $x + y + z$ C) 0 D) $2x - 4y + 3z$
55. If $x = 5$ and $y = x + 7$ then $\sqrt{x^2 + y^2} = \dots\dots$ []
 A) 13 B) 17 C) 26 D) 65
56. How much is $a^4 + 4a^2b^2 + b^4$ more than $a^4 - 8a^2b^2 + b^4 \dots\dots$ []
 A) $12ab^2$ B) $12a^2b^2$ C) $2a^4 + 2b^4$ D) $12ab^2$
57. If $ab = 1$ then $\frac{1}{1+a^{-1}} + \frac{1}{1+b^{-1}} = \dots\dots$ []
 A) 0 B) $a + b$ C) 1 D) $a - b$
58. If $a + b + c = 22$ and $ab + bc + ca = 54$ then $a^2 + b^2 + c^2 = \dots\dots$ []
 A) 376 B) 75 C) 377 D) 375
59. If $a + b = 11$ and $ab = 30$ then the value of $a - b = \dots\dots$ []
 A) -1 B) 2 C) 330 D) 19

60. The expression $\frac{bx(a^2x^2 + 2a^2y^2 + b^2y^2) + ay(a^2x^2 + 2b^2x^2 + b^2y^2)}{(ax + by)^2}$ is equal to ...[]
- A) $a(x + y)$ B) $bx + ay$ C) $ax + by$ D) $b(x + y)$
61. If $C = \frac{x-a}{x-b}$ then $x = \dots\dots\dots$ []
- A) $\frac{bc-a}{c-1}$ B) $\frac{c-a}{c-b}$ C) $\frac{c+a}{c+b}$ D) $\frac{1-c}{a-bc}$
62. If $\frac{x-a}{b+c} + \frac{x-b}{c+a} + \frac{x-c}{a+b} = 3$ then $x = \dots\dots\dots$ []
- A) abc B) $\frac{1}{abc}$ C) $a + b + c$ D) $\frac{1}{a + b + c}$
63. The value of 'm' for which $5^m = 125$ is []
- A) 3 B) 6 C) 2 D) 4
64. The degree of the expression $p^3q^3 + p^2q + p^4$ is []
- A) 4 B) 5 C) 6 D) 3
65. The perimeter of a square is $6x + 2$ then the side = []
- A) $\frac{3}{2}x + \frac{1}{2}$ B) $3x + 1$ C) $\frac{1}{2}x + 1$ D) $24x + 8$
66. Which one of the following is not a factor of $x^2 - 1 = \dots\dots\dots$ []
- A) $x - 1$ B) $x + 1$ C) 2 D) $x^2 - 1$
67. Solve $\frac{y+6}{5} = 14 + y$ is []
- A) -14 B) 16 C) -16 D) 5
68. The value of $\left[\left(-\frac{2}{3} \right)^{-2} \right]^2 = \dots\dots\dots$ []
- A) $5\frac{1}{6}$ B) $5\frac{1}{16}$ C) $4\frac{7}{16}$ D) $\frac{9}{16}$
69. If one factor of $6x^2 - x - 15$ is $(2x + 3)$ then the other factor is []
- A) $3x - 5$ B) $3x + 5$ C) $5x - 3$ D) $2x - 3$
70. If $x^2 + 6x + 9 = (x + 3)(x - a)$ then $a = \dots\dots\dots$ []
- A) -3 B) 3 C) 2 D) -2
71. The value of $(0.9)^2 + (0.1)^2 + 2(0.09) = \dots\dots\dots$ []
- A) .1 B) .9 C) 2 D) 1
72. If $13^x = 169$ then $x = \dots\dots\dots$ []
- A) 2 B) 13 C) 3 D) 1
73. If $1^3 + 2^3 + 3^3 + 4^3 = x^2$ then $x = \dots\dots\dots$ []
- A) 5 B) 7 C) 10 D) 15

74. The sixth triangular number is []
 A) 6 B) 10 C) 21 D) 28
75. If $x + y = 8$ and $xy = 15$ then $x^2 + y^2 =$ []
 A) 32 B) 34 C) 36 D) 38
76. The value of $\frac{6.4 \times 6.4 - 3.6 \times 3.6}{6.4 - 3.6} =$ []
 A) 10 B) 2.8 C) 12 D) 4.6
77. If $A = 5m + 11n - 15p$, $B = 12m - 13n + 19p$, $C = 7m - 6n + 21p$ then the value of $A+B-C$ is []
 A) $24m - 8n + 25p$ B) $10m + 4n - 17p$ C) $17m - 2n + 4p$ D) $12m + 5n - 4p$
78. The factors of $x^2 - 2y + xy - 4$ are []
 A) $(x - 2)$ and $(x + 2 + y)$ B) $(x + 2)$ and $(x + 2 - y)$
 C) $(x - y)$ and $(x + 2y - 1)$ D) $(x + y)$ and $(x - 2y + 2)$
79. $(2x + 3y)^2 - (2x - 3y)^2 =$ []
 A) $6xy$ B) $12xy$ C) $18xy$ D) $24xy$
80. If n is a positive integer then $n(n+1)(n+2)$ is []
 A) Even only when 'n' is even B) Even only when 'n' is odd
 C) Odd whenever 'n' is odd D) Divisible by 4 whenever 'n' is even

LEVEL - 2

1. The simplest form of $1 - \frac{1}{1 + \frac{a}{1-a}}$ is []
 A) a if $a \neq 0$ B) 1 C) a if $a \neq -1$ D) None
2. $a^x = c^q = b$ then $c^y = a^z = d$ then []
 A) $xy = qz$ B) $\frac{x}{y} = \frac{q}{z}$ C) $x + y = q + z$ D) $x - y = q - z$
3. $\frac{1}{1+a^{n-m}} + \frac{1}{1+a^{m-n}} =$ []
 A) 0 B) $\frac{1}{2}$ C) 1 D) a^{m+n}
4. $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} =$ []
 A) 0 B) 1 C) x^{a-b-c} D) x^{a+b+c}
5. If $lmn = 1$ then $\frac{1}{1+l+m^{-1}} + \frac{1}{1+m+n^{-1}} + \frac{1}{1+n+l^{-1}} =$ []
 A) lmn B) $l + m + n$ C) 1 D) 0

6. The length of a room is $9.0085 \times 10^{-2} \text{ cm}$. Which number is equivalent to this length []
 A) 0.90085 cm B) 0.090085 cm C) 900.85 cm D) 9008.5 cm
7. $\sqrt{\frac{8^{10} + 4^{10}}{64^2 + 4^9 \times 16}} = \dots\dots$ []
 A) 8 B) 16 C) 36 D) 256
8. If $a^x = \sqrt{b}$, $b^y = \sqrt[3]{c}$ and $c^z = \sqrt{a}$ then $xyz = \dots\dots$ []
 A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{6}$ D) $\frac{1}{12}$
9. The number to be multiplied by $(-7)^{-1}$ to get $(10)^{-1}$ as the product []
 A) $\frac{-7}{10}$ B) $\frac{-10}{7}$ C) $\frac{7}{10}$ D) $\frac{10}{7}$
10. If $4^x - 4^{x-1} = 24$ then the value of $2x$ will be []
 A) 5 B) 4 C) 3 D) 2
11. $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \dots\dots$ []
 A) 5^2 B) 5^3 C) 5^4 D) 5^5
12. If $\frac{a+b-c}{c} = \frac{a-b+c}{b} = \frac{-a+b+c}{a} = 1$ then $\frac{(a+b)(b+c)(c+a)}{abc} = \dots\dots$ []
 A) 0 B) 1 C) 8 D) 12
13. If $7^{2n+1} \div 49 = 7^3$ then $n = \dots\dots$ []
 A) 1 B) 2 C) 7 D) 3
14. If $x = a \left(\frac{m^2 + n^2}{2mn} \right)^{\frac{1}{2}}$ then $\frac{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}}{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}} = \dots\dots$ []
 A) mn B) $m+n$ C) $m-n$ D) $\frac{m}{n}$
15. $\frac{\left(a + \frac{1}{b}\right)^x \cdot \left(a - \frac{1}{b}\right)^y}{\left(b + \frac{1}{a}\right)^x \cdot \left(b - \frac{1}{a}\right)^y} = \dots\dots$ []
 A) $(ab)^{xy}$ B) $\left(\frac{a}{b}\right)^{xy}$ C) $\left(\frac{a}{b}\right)^{x+y}$ D) $\left(\frac{a}{b}\right)^{x-y}$
16. $(x^{m+n})^{n-n} \cdot (x^{n+p})^{n-p} \cdot (x^{p+m})^{p-m} = \dots\dots$ []
 A) 0 B) 1 C) $p+m$ D) mnp
17. $\sqrt[n]{(512)^n} = 64$ then $\frac{m}{n} = \dots\dots$ []
 A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) $\frac{9}{2}$ D) $\frac{3}{4}$
18. $\sqrt[mn]{(x^n)^m} = 0.2$ then $x^3 = \dots\dots$ []
 A) 0.04 B) 0.08 C) 0.008 D) 0.0016

19. $\frac{36^{\frac{7}{2}} - 36^{\frac{9}{2}}}{36^{\frac{5}{2}}} = \dots\dots$ []
 A) 1260 B) 1296 C) -1260 D) -1296
20. If $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}} = 0$ and $(a+b+c)^3 = K abc$ then $K = \dots\dots$ []
 A) 3 B) 9 C) 27 D) 81
21. If $a+b+c=0$ then $x^{\frac{a^2}{bc}} \cdot x^{\frac{b^2}{ac}} \cdot x^{\frac{c^2}{ab}} = \dots\dots$ []
 A) 2 B) x^3 C) x^{abc} D) $x^{a^2b^2c^2}$
22. If $x = 2\sqrt{2} + 7$ then $x + \frac{1}{x} = \dots\dots$ []
 A) $4\sqrt{2}$ B) $2\sqrt{3}$ C) 7 D) 1
23. If $a = x + \sqrt{x^2 + 1}$ then $x = \dots\dots$ []
 A) $\frac{1}{2}\left(a - \frac{1}{a}\right)$ B) $\frac{1}{2}\left(a + \frac{1}{a}\right)$ C) $2\left(a - \frac{1}{a}\right)$ D) $2\left(a + \frac{1}{a}\right)$
24. If $x = 7 + 4\sqrt{3}$ and $xy = 1$ then $\frac{1}{x^2} + \frac{1}{y^2} = \dots\dots$ []
 A) 64 B) 134 C) 194 D) $\frac{1}{49}$
25. If $x - \frac{1}{x} = a$ then the value of $x^3 - \frac{1}{x^3} = \dots\dots$ []
 A) a^3 B) $3a$ C) $a^3 - 3a$ D) $a^3 + 3a$
26. $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(a-c)^2}{(a-b)(b-c)} = \dots\dots$ []
 A) $3abc$ B) $3(a-b)(b-c)(c-a)$ C) 0 D) 1
27. If $a+b+c=0$ then $\frac{1}{b^2+c^2-a^2} + \frac{1}{c^2+a^2-b^2} + \frac{1}{a^2+b^2-c^2} = \dots\dots$ []
 A) abc B) $\frac{1}{abc}$ C) 0 D) 1
28. If $x+y+z=0$ then $\frac{(x+y)(y+z)(z+x)}{xyz} = \dots\dots$ []
 A) -1 B) 1 C) 0 D) $3xyz$
29. If $x+y=2z$ then $\frac{x}{x-z} + \frac{z}{y-z} = \dots\dots$ []
 A) $1+x$ B) yz C) 1 D) xyz
30. If $x + \frac{1}{y} = 1$ and $y + \frac{1}{z} = 1$ then $z + \frac{1}{x} = \dots\dots$ []
 A) xyz B) 0 C) $\frac{x}{y}$ D) 1

31. If $a=b=c$ then $\frac{(a+b+c)^2}{a^2+b^2+c^2} = \dots\dots$ []
 A) 1 B) 2 C) 3 D) 4
32. $\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] = \dots\dots$ []
 A) $a^3+b^3+c^3+3abc$ B) $a^3+b^3+c^3-3abc$
 C) $a^3+b^3+c^3+3abc(a+b+c)$ D) $3abc$
33. If $a+b+c=9$ and $a^2+b^2+c^2=29$ then the value of $a^3+b^3+c^3-3abc = \dots\dots$ []
 A) 9 B) 3 C) 27 D) 81
34. If $x=a(b-c), y=b(c-a)$ and $z=c(a-b)$ then $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 = \dots\dots$ []
 A) $\frac{3xyz}{abc}$ B) $\frac{xyz}{abc}$ C) 1 D) $3xyzabc$
35. If $a=1$ then the value of $15a^3 - (3a^3 - 1) - (4a^4 + a^3 - 3) - (a^3 - 1) = \dots\dots$ []
 A) 11 B) 1 C) 10 D) 17
36. $\frac{(83 \times 83 \times 83) + (27 \times 27 \times 27)}{(83 \times 83) - (83 \times 27) + (27 \times 27)} = \dots\dots$ []
 A) 166 B) 110 C) 56 D) 54

LEVEL - 3

1. If $x^2+1=2x$ then $x^7+x^5+x^4+x^{-9}+x^{-8} = \dots\dots$ []
 A) 0 B) 4 C) 5 D) 8
2. If $2^x + 2^{x+1} + 2^{x+2} + \dots\dots + 2^{x+7} = 255$ then $x = \dots\dots$ []
 A) 0 B) 2 C) -1 D) 1
3. $\frac{154}{69} = a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}}$ then $(c+d) - (a+b) = \dots\dots$ []
 A) 4 B) 5 C) 6 D) 2
4. If $x+y+z=0$ then $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = \dots\dots$ []
 A) $\sqrt{3}$ B) $3\sqrt{3}$ C) 3 D) 0
5. $2^n - 2^{n-1} = 4$ then $n^n = \dots\dots$ []
 A) 1 B) $\frac{3}{2}$ C) 2 D) 27
6. $\frac{(3.63)^2 - (2.37)^2}{3.63 - 2.37} = \dots\dots$ []
 A) 6 B) 1.36 C) 2.26 D) 1.26

7. $(x^6 \cdot y^{-\frac{5}{4}})^{\frac{4}{3}} = \dots\dots\dots$ []
 A) $x^{-24}y$ B) $x^{-8}y^{\frac{5}{3}}$ B) $x^8 \cdot y^{\frac{-5}{3}}$ D) $x^{-8} \cdot y^{\frac{-5}{3}}$
8. If $\left(\frac{7}{5}\right)^{-8} \times \left(\frac{7}{5}\right)^{-12} = \left(\frac{7}{5}\right)^{5x}$ then $x = \dots\dots\dots$ []
 A) 5 B) -5 C) 4 D) -4
9. a, b, c are three positive real numbers $a(b+c)=44$, $b(c+a)=50$, $c(a+b)=54$ then $abc \dots\dots$ []
 A) 210 B) 120 C) 126 D) 156
10. If $x=5$ then $(x+2)(2x-1)(2x+1)(3x-4) = \dots\dots\dots$ []
 A) 0 B) 124 C) 168 D) 184
11. a, b, c are three real numbers $a-7b+8c=4$ and $8a+4b-c=7$ then $a^2-b^2+c^2 = \dots\dots\dots$ []
 A) 4 B) 1 C) 5 D) 11
12. a, b, c are real numbers $a+b+c=7$, $a^2+b^2+c^2=35$, $a^3+b^3+c^3=151$ then the value of $abc = \dots\dots\dots$ []
 A) -15 B) 8 C) 9 D) 115
13. If $(a-1)^2 + (b-2)^2 + (c-3)^2 + (d-4)^2 = 0$ then $abcd + 1 = \dots\dots\dots$ []
 A) 0 B) 1 C) 25 D) 24
14. If $a^2-b^2=21$ and $a^2+b^2=29$ then $ab = \dots\dots\dots$ []
 A) 10 B) 0 C) 25 D) 16
15. If a, b, c are three prime numbers, $a.b.c. = 17(a+b+c)$ then the values of a, b, c are []
 A) 19, 07, 2 B) 17, 09, 2 C) 13, 19, 17 D) 2, 17, 19
16. If $a^3+3a^2+3a+2=0$ then $(a+1)^{2018} + (a+1)^{2019} + (a+1)^{2020} = \dots\dots\dots$ []
 A) 0 B) 1 C) 2 D) -1
17. a, b, c are real number such that $a^2+4b^2+16c^2=48$ and $ab+4bc+2ca=24$ then $a+2b+4c = \dots\dots\dots$ []
 A) 144 B) 12 C) 121 D) 11
18. $\frac{(2^4)^6}{(4^6)^2} = \dots\dots\dots$ []
 A) 20 B) 32 C) 1 D) 8
19. The sum of three numbers is 150. The second number is thrice the first and the third is 10 less than twice the second then the second number $\dots\dots\dots$ []
 A) 16 B) 36 C) 86 D) 48
20. The sum of two numbers is 7. If one number is subtracted from the other the result is 5 then the number are $\dots\dots\dots$ []
 A) 6, 3 B) 6, 1 C) 1, 7 D) 8, 1
21. If $x^2+xy=20$ and $y^2+xy=30$ then $xy = \dots\dots\dots$ []
 A) 8 B) 10 C) 12 D) 14
22. If $3x-y=12$ then $\frac{8^x}{2^y} = \dots\dots\dots$ []
 A) 2^{12} B) 12^2 C) 3^{12} D) 16

23. If $\sqrt{x-3} = y$ and $\sqrt{y-4} = z$ and $\sqrt{z-5} = 2$ then $x = \dots$ []
 A) 6129 B) 6565 C) 7103 D) 7228
24. If $\frac{x}{y+z} = \frac{y}{z+x} = \frac{z}{x+y}$ then each fraction will be []
 A) $\frac{1}{2}$ B) -1 C) $\frac{1}{2}$ or -1 D) $\frac{1}{2}$ or 1
25. If $a+b=123$, $b+c=37$, $c+a=40$ then $\sqrt{a+b+c} = \dots$ []
 A) 30 B) 20 C) 10 D) 18
26. If $x^{\sqrt{x}} = (\sqrt{x})^x$ then $x = \dots$ []
 A) 2 B) 3 C) 4 D) 1
27. The value of $\frac{2^{n+2} - 2^{n+1}}{4 \times 2^{n-1}} = \dots$ []
 A) 1 B) 2 C) 3 D) 4
28. $x^a + 1$ is factor of $x^{ab} + 1$ if b is []
 A) Odd number B) Even number C) Negative number D) Radical
29. $(-2.56 \times 4.2) = \dots$ []
 A) 0 B) 1 C) 10.752 D) -10.752
30. The algebraic expression of double the sum of one third of "a" and 'm' is []
 A) $\frac{1}{3}(2a+m)$ B) $2 + \left(\frac{1}{3}a+m\right)$ C) $\frac{2}{3}a+m$ D) $2\left(\frac{1}{3}a+m\right)$
31. If one-third of one-fourth of one seventh of a number is 20. Then the number []
 A) 420 B) 240 C) 1680 D) 560
32. The value of $\left(\frac{a^{11}}{a^{13}}\right)^{\frac{1}{143}} \times \left(\frac{a^{13}}{a^7}\right)^{\frac{1}{91}} \times \left(\frac{a^7}{a^{11}}\right)^{\frac{1}{77}} = \dots$ []
 A) a B) $\frac{1}{a}$ C) 1 D) -1
33. The value of $2^{\frac{5}{2}} - 2^{\frac{3}{2}} = \dots$ []
 A) $2\sqrt{2}$ B) $3\sqrt{2}$ C) $5\sqrt{2}$ D) $3\sqrt[3]{2}$
34. $12 \div [2(6-7+4)] \times 2 = \dots$ []
 A) 2 B) 4 C) 6 D) 8
35. $12 \div 2(6-7+4) \times 2 = \dots$ []
 A) 2 B) 4 C) 36 D) 18
36. If $2(3+2x) - 5(x-1) = 12$ then $x = \dots$ []
 A) -3 B) -2 C) -1 D) 1
37. If $5^x = 1000$ then the value of 5^{x-2} is []
 A) 10 B) 20 C) 80 D) 40
38. $1+3+5+7+9+11+13+15+17+19+21+23+25 = \dots$ []
 A) 13^2 B) 14^2 C) 11^2 D) 12^2

LEVEL - 1. ANSWERS

1. B	16. B	31. B	46. C	61. A	76. A
2. D	17. D	32. A	47. B	62. C	77. B
3. A	18. A	33. B	48. B	63. A	78. A
4. C	19. D	34. B	49. C	64. C	79. D
5. D	20. B	35. C	50. A	65. A	80. D
6. A	21. C	36. B	51. C	66. C	
7. D	22. C	37. D	52. B	67. C	
8. B	23. B	38. D	53. D	68. A	
9. C	24. D	39. C	54. C	69. A	
10. B	25. B	40. C	55. A	70. A	
11. B	26. D	41. A	56. B	71. D	
12. D	27. C	42. C	57. C	72. A	
13. B	28. C	43. D	58. A	73. C	
14. A	29. A	44. A	59. A	74. C	
15. C	30. D	45. A	60. B	75. B	

LEVEL - 2. ANSWERS

1. A	10. A	19. C	28. A
2. A	11. D	20. C	29. C
3. C	12. C	21. B	30. D
4. B	13. B	22. A	31. C
5. C	14. D	23. A	32. B
6. B	15. C	24. C	33. C
7. B	16. B	25. D	34. A
8. D	17. B	26. C	35. A
9. A	18. C	27. C	36. B

LEVEL - 3. ANSWERS

1. C	11. B	21. C	31. C
2. A	12. A	22. A	32. C
3. D	13. C	23. D	33. A
4. C	14. A	24. C	34. B
5. D	15. D	25. C	35. C
6. A	16. B	26. C	36. C
7. B	17. B	27. A	37. D
8. D	18. C	28. A	38. A
9. B	19. D	29. B	
10. D	20. B	30. D	